Minimal Compensation and Incentives for Effort

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Abstract

When does paying a strictly positive compensation in every state of the world improve incentives to exert effort? I show that in the typical model of moral hazard it happens only when the effort is a strict complement to consumption. If the cost of effort is monetary, a positive minimal compensation strengthens incentives only when the agent is prudent and always does so when the marginal utility of consumption is unbounded at zero consumption. I suggest applications of these results in the personal income taxation.

1 Introduction

The model of moral hazard demonstrates the trade-off between insurance and incentives. A risk neutral principal wants to motivate a risk averse agent to exert effort. Moreover, the principal needs to provide the agent with some minimal level of utility, e.g. due to the agent's participation decision. The trade-off exists, since the efficient provision of utility requires the full insurance of the agent, which undermines any incentives for effort. In this paper I show that this trade-off is not absolute: sometimes increasing insurance benefits incentives. I identify cases in which the optimal compensation of the agent includes a positive unconditional pay, even though the principal is not obliged to provide the agent with any minimal level of utility. Thus, the unconditional pay plays a role of the incentive pay, as it strengthens the agent's willingness to exert effort.

In my framework the agent chooses whether to exert effort or not, which affects the distribution of output. The principal, who observes only the realized output, sets up a compensation scheme to motivate the agent to exert effort at the lowest cost. The principal is constrained only by incentive compatibility - the agent needs to be better off by exerting effort. I impose no participation or individual rationality constraints. The sole role of the compensation scheme is to provide incentives for effort.

I study when the optimal compensation scheme includes a positive minimal compensation regardless of the realized output.[1] First, a positive minimal pay is optimal only if effort is a complement to consumption. Only then higher consumption reduces the cost of effort and relaxes the incentive

[1] Hölmstrom (1979) shows that the optimal compensation should vary with any observable variable that is informative of the agents' level of effort. The unconditional pay does not violate this informativeness principle as long as there is a state-contingent bonus on top of it.

compatibility constraint. Consecutively, I focus on the classical case of complementarity between consumption and effort - the model with a monetary cost of effort. When the output distribution is sufficiently rich,[2] the agent will be compensated in every state of the world only if he is prudent, i.e. only when the marginal utility of consumption is convex. Without prudence, paying the agent in all the states that are more likely without effort undermines incentives. Finally, a sufficient condition for a positive minimal pay for an arbitrary distribution of output is an unbounded marginal utility of consumption at 0. This simple condition means that marginally increasing the agent compensation above zero always raises the expected utility from exerting effort more strongly than the expected utility from shirking.

Grossman and Hart (1983) study various features of the optimal compensation scheme in the moral hazard problem, such as monotonicity and concavity with respect to output realization. My paper is concerned with the particular feature: the minimal compensation level. Mirrlees (1999) provide conditions under which the first-best outcome to be approximated with a step function with two compensation level. As the lower compensation level converges to zero, the agent's increased effort makes realization of the low pay unlikely. I characterize the polar case, in which the minimal payment to the agent is optimally bounded away from zero. Holmstrom and Milgrom (1991) propose another environment, based on multitasking, in which insurance is good for incentives. Compensation which depends on observed outcomes makes the agent shift the effort away from tasks with unobserved outcomes. As a result, the optimal contract may specify a fix wage which does not depend on the observed outcomes. In my paper, I show that a certain amount of insurance can improve incentives in the standard model with a single task.

I discuss the application of my results in the design of the optimal tax systems. Effort can be interpreted either as an investment in a risky venture or a costly education decision which affects future distribution of income. When the marginal utility of consumption is unbounded at zero, taxing the high income agents and providing positive transfer to the low income agents actually improves incentives for effort. The minimal compensation can be understood as a basic income - an unconditional cash transfer to any agent. Van Parijs (1991) justifies the basic income on the moral grounds. I provide conditions under which the basic income has a positive impact on incentives and can be justified on the efficiency grounds.

Structure of the paper. The next section introduces the framework. Section 3 presents the main theoretical results. They are illustrated by the numerical exercise in Section 4. The consecutive section proposes the application of the theory in taxation. The last section concludes and discusses possible extensions.

2 Model

The agent chooses whether to exert effort ($e = 1$) or not ($e = 0$).[3] The effort affects the distribution of output, which has a finite support $Y \subset \mathbb{R}_+$ and the probability mass function $p_e : Y \to [0, 1]$.

[2] There are at least two output levels which are more likely without effort.

[3] I focus on the binary effort decision, which simplifies the analysis as I need to consider only one incentive-compatibility constraint. I discuss the extension to the case of continuous effort in the last section.

The agent's flow utility function $U(c, e) : \mathbb{R}_+ \times \{0, 1\} \to \mathbb{R}$ is increasing, strictly concave and twice differentiable in consumption c. The effort is costly: $U(c, 0) - U(c, 1) > 0$ for all $c > 0$, and I assume that this difference is strictly positive in the limit as $c \to 0$.[4]

The principal does not observe the effort and compensates the agent with payments $w : Y \to \mathbb{R}_+$ which depend only on the realized output. The optimal compensation scheme solves

$$\max_{w:Y\to\mathbb{R}_+} \sum_{y\in Y} p_1(y)(y - w(y))$$

subject to the incentive compatibility constraint, guaranteeing that the agent is better off by exert effort

$$\sum_{y\in Y} p_1(y)U(w(y), 1) \geq \sum_{y\in Y} p_0(y)U(w(y), 0). \tag{IC}$$

Note that the agent does not make a participation decision, nor is the principal commited to provide the agent with any minimal level of utility. The only role of the compensation scheme w is to provide agent with incentives for effort. I assume that there exists a compensation scheme which implements a positive effort.[5] Under this assumption, the principal always prefers to motivate effort if the difference in expected output with and without effort is sufficiently high. I assume that this is the case.

3 When is the minimal compensation strictly positive?

The three propositions below characterize conditions under which a positive minimal compensation is optimal.

Proposition 1. *The minimal compensation is zero if (i) there exists an outcome realization that is possible only when effort is absent or (ii) effort is a substitute to consumption:* $\forall_{c>0} U_c(c, 0) \geq U_c(c, 1)$.

Proof. It is optimal to pay the agent only if it relaxes the incentive compatibility constraint. Suppose that there is $\bar{y} \in Y$ which is possible only without effort: $p_1(\bar{y}) = 0$ and $p_0(\bar{y}) > 0$. Increasing $w(\bar{y})$ always tightens (IC), so optimally $w(\bar{y}) = 0$.

Denote the marginal utility from consumption by $U_c(c, e)$. The optimal contract involves positive compensation $w(y) > 0$ for some output $y \in Y$ only if

$$U_c(w(y), 1) - \frac{p_0(y)}{p_1(y)} U_c(w(y), 0) = \frac{1}{\mu} \tag{1}$$

[4]Otherwise, the agent that receives no compensation at all is indifferent between the two levels of effort and the optimal compensation is trivially equal to 0.

[5]This assumption may be wrong if the effort cost is sufficiently high. Suppose that $Y = \{\underline{y}, \bar{y}\}$, $p_1(\bar{y}) = p, p_1(\underline{y}) = 1 - p, p_0(\bar{y}) = 0, p_0(\underline{y}) = 1$. With the utility function $U(c, e) = -e^{-\gamma(c+(1-e)\epsilon)}$ the incentive compatibility constraint (IC) can be satisfied only if $\gamma\epsilon < -\log(1 - p)$.

where $\mu > 0$ is the Lagrange multiplier of the incentive constraint. On the other hand, $w(y) = 0$ can be optimal only if

$$U_c(0,1) - \frac{p_0(y)}{p_1(y)} U_c(0,0) \leq \frac{1}{\mu}. \tag{2}$$

If $\forall_{c>0} U_c(c,0) \geq U_c(c,1)$ then the left-hand side of (1) is negative for any output realization that is more likely without effort. Hence, if $p_0(y) > p_1(y)$, which is true for at least one $y \in Y$, then optimally $w(y) = 0$. □

The agent should not be compensated for output which unambiguously identifies the missing effort. This result is closely related to the 'unpleasant theorem' of Mirrlees (1999), according to which the principal can motivate the agent by introducing severe punishments for output levels that are unlikely under the positive effort. Furthermore, the principal will not use the positive minimal pay when consumption is a substitute to effort. When consumption and effort are substitutes, the utility cost of effort $U(c,0) - U(c,1)$ weakly increases with consumption. In order to keep the expected cost of effort low, the principal will pay the agent only for output levels which coincide with the positive effort. Specifically, with the commonly assumed additively separable disutility of effort the optimal contract always involve zero minimal pay.

Proposition 1 shows that the complementarity between consumption and effort is required for the positive minimal pay. In the remaining part of the paper I derive a sharper characterization of the minimal pay under the classical case of such complementarity - a situation in which the effort has a purely monetary cost.

Assumption 1. *The utility function is $U(c,e) = u(c + (1-e)\epsilon)$, where $u \in \mathcal{C}^3$ is increasing and strictly concave and $\epsilon > 0$. Moreover, no output realization unambiguously identifies the missing effort: $p_0(y) > 0 \implies p_1(y) > 0$.*

Under Assumption 1 the effort has a fixed monetary cost $\epsilon > 0$. Hence, by shirking and not incurring the cost, the agent can increase his consumption.

Proposition 2. *Under Assumption 1, the minimal compensation is zero if any of the following conditions hold:*

1. *The utility function satisfies $u'''(c) \leq 0$ for all $c > 0$ and there are at least two output levels that are more or equally likely without effort.*

2. *The utility function u has a constant absolute risk aversion.*

Proof. [*1.*] First I will derive an additional necessary optimality condition. Take two output realizations $y, y' \in Y$ such that both $w(y)$ and $w(y')$ satisfy (1) and at least one of them is positive. Perturb $w(y)$ by a small δ and $w(y')$ by $-\frac{p_1(y)}{p_1(y')}\delta$. This perturbation does not affect the principal's profit if the effort is unchanged. Define $V_{\tilde{y}}(w) \equiv u(w) - \frac{p_0(\tilde{y})}{p_1(\tilde{y})} u(w+\epsilon)$. The impact of the perturbation on (*IC*), taking into consideration the terms up to the second order, is

$$\delta \left(p_1(y) V'_y(w(y)) - \frac{p_1(y)}{p_1(y')} p_1(y') V'_{y'}(w(y')) \right) + \frac{\delta^2}{2} \left(p_1(y) V''_y(w(y)) + \left(\frac{p_1(y)}{p_1(y')} \right)^2 p_1(y') V''_{y'}(w(y')) \right).$$

Optimality requires that this expression is non-positive, since otherwise it would be possible to relax the incentive-compatibility constraint without losses in profits. The first-order component is zero by the necessary condition (1). Hence, the optimal contract satisfies

$$p_1(y')V''_y(w(y)) + p_1(y)V''_{y'}(w(y')) \leq 0. \tag{3}$$

Now, take any $\bar{y} \in Y$ such that $\frac{p_0(\bar{y})}{p_1(\bar{y})} \geq 1$. When $u''' \leq 0$, we have $1 \geq \frac{u''(w(\bar{y}))}{u''(w(\bar{y})+\epsilon)}$, which together implies that $V''_{\bar{y}}(w(\bar{y})) \geq 0$. If there are two such output levels, then they violate the necessary optimality condition (3) unless compensation in both states is 0 or (1) holds for at most one of them. Either way, for at least one of these output levels the compensation is optimally 0.

[*2.*] Suppose that the utility is CARA ($u(c) \equiv -e^{-\gamma c}$) and that the incentive constraint holds as equality for some compensation scheme w with a positive minimal pay $\underline{w}$. Note that

$$\sum_{y \in Y} p_1(y)e^{-\gamma w(y)} = \sum_{y \in Y} p_0(y)e^{-\gamma(w(y)+\epsilon)} \implies \sum_{y \in Y} p_1(y)e^{-\gamma(w(y)-\underline{w})} = \sum_{y \in Y} p_0(y)e^{-\gamma(w(y)-\underline{w}+\epsilon)},$$

so the principal can save resources by uniformly reducing the compensation in every contingency. □

When the output distribution is sufficiently rich - there are at least two output levels that are less likely with the positive effort - prudence ($u''' > 0$) becomes the necessary condition for the positive minimal pay. Without prudence, any contract that satisfies (1) at each output level violates the second order condition for the local maximum. The principal's problem is convex in compensation for the output levels which are less likely under effort. Then it is optimal to keep the compensation positive for at most one of these output levels. Rothschild and Stiglitz (1971) show that prudent individuals save more when faced with more risk. The effort decision resembles the savings decision, as effort, besides changing the distribution of output, reduces the agents consumption by a constant amount in each contingency. Prudent agents, when faced with less income risk because of the higher minimal compensation, are willing to save less, i.e. exert more effort. This analogy, however, has its limits. The precautionary saving motive increases in the absolute prudence $-u'''/u''$, as demonstrated by Kimball (1990). Nevertheless, even an arbitrarily high level of the absolute prudence is not sufficient to guarantee the optimum with a positive minimal compensation. The CARA utility function, for which the absolute prudence equals the absolute risk aversion, always involves the lowest compensation of zero. In this case, since the agent preferences over lotteries are independent of wealth, providing an unconditional income does not affect the incentive constraint.

Proposition 3. *Suppose that Assumption 1 holds. The minimal compensation is positive* ***if*** $\lim_{c\to 0} u'(c) = +\infty$*. When the utility function has non-increasing absolute risk aversion, the minimal compensation is positive for an arbitrary distribution of output* ***only if*** $\lim_{c\to 0} u'(c) = +\infty$*.*

Proof. The necessary condition for the corner solution (2) is never satisfied when $\lim_{c\to 0} u'(c) = +\infty$, which proves the 'if' part. To prove the 'only if' part, suppose that $u'(0)$ is finite and construct a distribution of output with some $\bar{y} \in Y$ such that $\frac{p_0(\bar{y})}{p_1(\bar{y})} = \frac{u'(0)}{u'(\epsilon)}$. When the absolute risk aversion is non-increasing, $\frac{u'(c)}{u'(c+\epsilon)}$ is non-increasing with c, since $\frac{\partial}{\partial c}\frac{u'(c)}{u'(c+\epsilon)} = (a(c+\epsilon) - a(\epsilon))\frac{u'(c)}{u'(c+\epsilon)}$, where

$a(c) \equiv -\frac{u''(c)}{u'(c)}$ stands for the absolute risk aversion. Hence, for any $w(\bar{y}) \geq 0$ the left-hand side of (1) is non-positive and the only possible solution lays in the corner with $w(\bar{y}) = 0$. □

An unbounded marginal utility from consumption at zero is a sufficient condition for a positive minimal pay. The shirking agent has higher consumption, since he does not incur the cost ϵ. When $\lim_{c\to 0} u'(c) = +\infty$ and compensation is zero at some output level, the differences in marginal utilities with and without effort dominate any possible difference in odds. As a result, a marginal increase in compensation for any output level, starting from 0, improves the agent's expected utility from exerting effort in comparison to shirking. In other words, the principal can decrease the utility cost of exerting effort $u(c+\epsilon) - u(c)$ by a large amount by marginally increasing the minimal compensation above zero. This result is apparent when $\lim_{c\to 0} u(c) = -\infty$: without the positive minimal compensation the expected utility of exerting effort is $-\infty$, while the expected utility of shirking is finite. However, the results holds also for utility functions taking finite values at zero consumption, e.g. CRRA utility with the relative risk aversion lower than 1.

Under plausible conditions, the unbounded marginal utility at zero becomes a necessary condition for a positive minimal pay for an arbitrary distribution of output. For this result to hold, we need a non-increasing absolute risk aversion. Individual preferences satisfy this realistic property if and only if the propensity to take risks does not decrease with wealth. If, on the contrary, the absolute risk aversion is increasing and the marginal utility of consumption is bounded, then we can always find a distribution of output which would imply a zero minimal compensation. Finally, note that Propositions 2 and 3 are consistent with each other, since an unbounded marginal utility at zero implies prudence in the neighborhood of zero consumption.[6]

4 Numerical example

Assumption 2. *The utility function is* $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, $\sigma > 0$. *There are two possible output realizations* $Y = \{0, \bar{y}\}$. *The high output realization is possible only under effort:* $p_1(\bar{y}) = p \in (0,1), p_0(\bar{y}) = 0$.

Lemma 1. *Under Assumption 2 the optimal contract is linear in the cost of effort, i.e. for any* σ *and* p *there exist* $\omega(\sigma,p)$ *and* $\beta(\sigma,p)$ *such that the optimal contract satisfies*

$$w(0) = \omega(\sigma,p)\epsilon, \quad w(\bar{y}) = w(0) + \beta(\sigma,p)\epsilon. \tag{4}$$

Proof. In the Appendix. □

Under simplifying Assumption 2 the optimal contract is linear in effort cost ϵ. The compensation can be described with two coefficients: ω, which stands for the guaranteed pay, and β, which stands for the bonus for the high output realization. I compare the optimal contract with the 'no insurance' contract, in which the agent is paid only when the high output is realized. In such

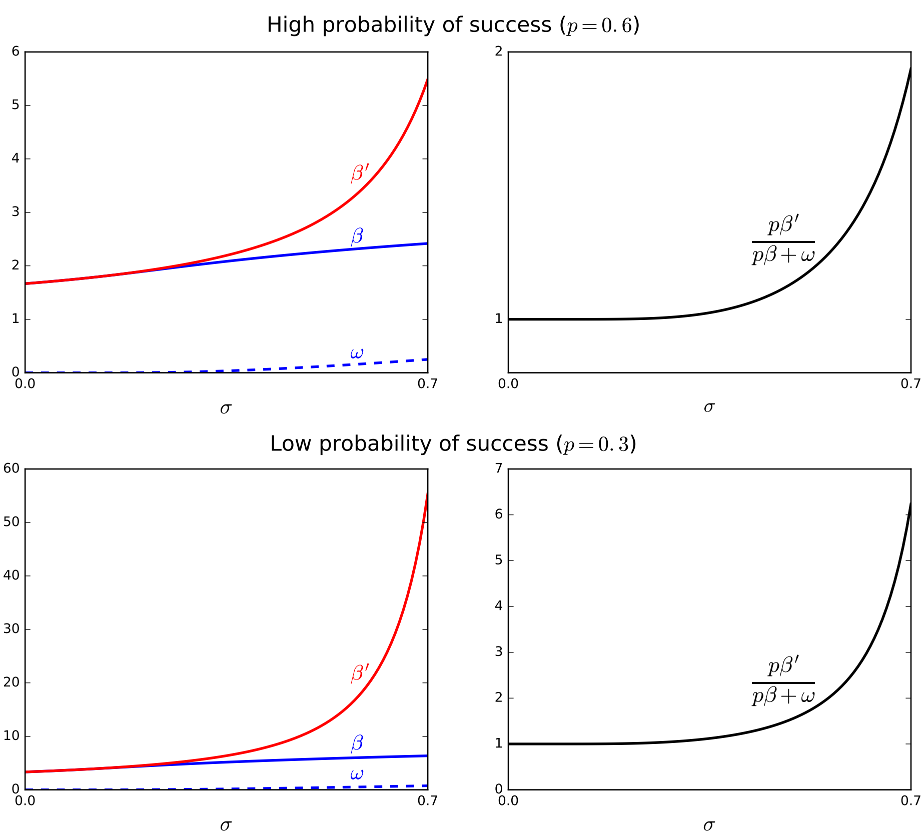
[6] Suppose that $u''' \leq 0$ at some interval $(0, \bar{c})$ with $\bar{c} > 0$. We can bound $u''(0)$ from below by $u''(\bar{c})$, which in turn allows us to bound $u'(0)$ from above by $u'(\bar{c}) - \bar{c}u''(\bar{c}) < \infty$.

contract the incentive provision requires that the agents receives $\beta'(\sigma, p)\epsilon$ in the high output state, where $\beta'(\sigma, p) \equiv p^{\frac{1}{\sigma-1}}$.

Figure 1 shows the optimal and 'no insurance' contracts for different values of relative risk aversion σ and probability of high output realization p. The parameter σ is kept below 1, because only then 'no insurance' contract can motivate effort. The rows correspond to different probabilities of the high output realization under effort. The left column presents the coefficients ω and β of the optimal contract and the coefficient β' of 'no insurance' contract. The right column shows the relative cost of providing incentives with 'no insurance' contract.

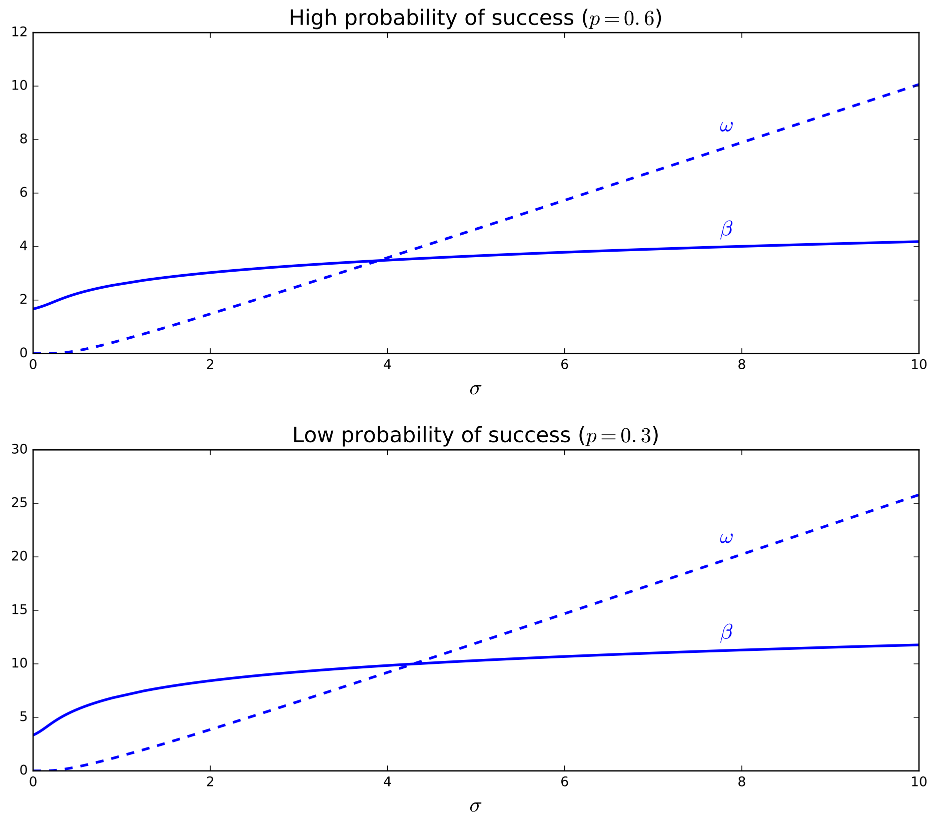
When the risk aversion is low, the minimal compensation is minuscule - the optimum is indistinguishable from 'no insurance' contract. As the risk aversion increases, both ω and β steadily rise. However, for 'no insurance' contract to provide the same incentives, β' has to grow much quicker. When σ approaches 1, β' diverges to $+\infty$, while ω and β remain finite. The relative cost of providing incentives without a guaranteed compensation is thus exploding. When the probability of high output realization is low, it is much harder to provide incentives for effort without insurance. As a result, β' and the cost gap between the two contracts increase even faster with the relative risk aversion.

Figure 1: Comparison of the optimal and 'no insurance' contracts



It is notable that a rather small minimal compensation can lead to huge differences in the cost of incentives. The exercise is conducted for low values of the relative risk aversion, since only then the comparison with 'no insurance' contract is possible. Figure 2 shows that for higher values of the risk aversion the guaranteed compensation ω can be substantial and exceed the value of bonus β.

Figure 2: The optimal contract for high levels of the risk aversion



5 Application to taxation

In this section I propose two interpretations of the theoretical framework studied above. In both environments there is a government that sets up an income tax with the sole aim of maximizing the tax revenue. Moreover, I assume that the utility functions of individuals feature the unbounded marginal utility at zero consumption.

Consider an entrepreneur endowed with wealth $\epsilon > 0$. The wealth can be either consumed or invested in a venture that is risky, yet profitable in expectations. This investment decision, which is unobserved by the government, affects the distribution of entrepreneur's income. Moreover, suppose that with positive probability venture fails to produce any value. In this case the government cannot tell apart the unlucky entrepreneurs from the agents that simply consumed their endowment. Nevertheless, by Proposition 3 providing transfers to entrepreneurs with no income improves incentives for risky investment and leads to higher tax revenue. Albanesi (2006) characterizes the optimal taxation of entrepreneurial income in the presence of moral hazard. However, she assumes that the cost of effort is additively separable from consumption, which by Proposition 1 precludes any incentive role of the positive transfer for unlucky entrepreneurs. It is natural to think that entrepreneurs' consumption is not independent of their investment choices, since the raised funds are frequently used to throw lavish startup parties.[7]

Alternatively, interpret the agent as an individual who considers going to college. The monetary cost of college ϵ becomes the sum of admission fees and foregone earnings. If both educated and uneducated workers face a possibility of zero labor productivity, which can be interpreted as chronic unemployment or disability, then by Proposition 3 transfers to workers with no earnings improve the incentives for education. Hence, when the return to education is sufficiently high, the sole incentive provision can justify a redistributive income tax even if the government cares only about the total tax revenue. Note that, similarly to Badel and Huggett (2014), I assume that the government cannot base its policies on the individual's education decision. An alternative approach, in which the government optimizes with respect to both the income tax and education subsidies, was explored by Bovenberg and Jacobs (2005) and Krueger and Ludwig (2013).

6 Conclusions and extensions

The relation between insurance and incentives is not necessarily monotone. Although no risk and full insurance precludes incentives, not always full risk and no insurance, i.e. paying the agent a constant fraction of output, implies the strongest incentives. I show that when effort and consumption are complements, increasing insurance by introducing an unconditional minimal pay can strengthen incentives for effort. When the cost of effort is monetary, it happens only when agents are prudent and always when the marginal utility of consumption is unbounded at zero consumption. I argue that these results are policy relevant. They highlight the efficiency role of unconditional cash transfers in encouraging a costly investment, be it an entrepreneurial activity or education.

In the remainder of this section I discuss two possible extensions. The assumption of binary effort simplifies the analysis, since we need to consider only a single incentive constraint. Some results generalize to the case of continuous effort. I will show that the 'if' part of Proposition 3 holds also in this case, namely: $lim_{c\to 0}u'(c) = \infty$ implies that a positive minimal is optimal. Suppose that the agent chooses the effort $e \in [0, 1]$. The effort affects the probability mass function $p_e(y)$, which is differentiable in effort at each output level. I assume that $p_e(y) \geq \underline{p} > 0$ for all $y \in Y$ and all effort levels $e \in [0, 1]$, which precludes the 'unpleasant theorem' of Mirrlees (1999). Suppose that the principal wants to implement the effort level $e^* > 0$. Then the agent's expected utility from exerting effort e is $\sum_y p_e(y)u(w(y) + (e^* - e)\epsilon)$.[8] Suppose that the marginal utility is unbounded

[7]'Going too big with the launch party' is the first entry on the list of common startup mistakes in Porges, S. (2013, May 17). The 10 PR Disasters All Startups Need To Avoid. *Forbes*. Retrieved from http://www.forbes.com.

[8]The principal, besides paying the compensation w, provides the agent with resources to cover the cost of effort.

at zero and that there is an output level $y \in Y$ with $w(y) = 0$. By marginally decreasing effort, the agent gains an unbounded amount in utility terms by avoiding the zero consumption, while loses at most a finite value due to the affected distribution of output. Hence, the unbounded marginal utility at zero consumption implies a positive minimal compensation.

The presented model is static. Spear and Srivastava (1987) express a dynamic moral hazard model with a promised-utility approach, where the agent's compensation consists of an immediate payoff and future utility promises, which need to be fulfilled by the principal. On the one hand, the dynamic problem of the principal involves additional promise-keeping constraints which can give raise to the positive minimal pay even without the incentive justification. On the other hand, the promise-keeping constraint in the moral hazard model is expressed as equality. The principal cannot provide neither less nor more utility than promised. If the promised utility along some output path is decreasing, it's likely that so will the minimal positive pay. Rogerson (1985) and Thomas and Worrall (1990) show that, with an additively separable disutility from effort and a utility from consumption which is unbounded below, the promised utility converges to $-\infty$ with probability 1. The investigation of the limiting behavior of the promised utility with complementarity between effort and consumption is an interesting research topic, however it is beyond the scope of this paper.

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Appendix: additional proofs

Proof of Lemma 1. Denote the inverse function of u with g and the inverse function of u' with h. We can express the bonus as a function of $w(0)$ with the (IC) constraint

$$w(\bar{y}) = g\left(\frac{1}{p}u(w(0)+\epsilon) - \frac{1-p}{p}u(w(0))\right). \tag{5}$$

By Proposition 3 we know that the minimal compensation is positive. We can use the interior optimality condition (1) with respect to $w(0)$ and $w(\bar{y})$ to obtain

$$w(\bar{y}) = h\left(u'(w(0)) - \frac{1}{1-p}u'(w(0)+\epsilon)\right). \tag{6}$$

Combining both equations and dividing by $w(0)$, we get

$$\left(\frac{1}{p}(1+\epsilon/w(0))^{1-\sigma} - \frac{1-p}{p}\right)^{\frac{1}{1-\sigma}} = \left(1 - \frac{1}{1-p}(1+\epsilon/w(0))^{-\sigma}\right)^{-\frac{1}{\sigma}}. \tag{7}$$

The equation above is affected by ϵ or $w(0)$ only through the ratio $\epsilon/w(0)$. It means that if we perturb ϵ and adjust $w(0)$ to keep the ratio constant, the equation will be satisfied. Hence, there exists $\omega(\sigma,p)$ such that $w(0) = \omega(\sigma,p)\epsilon$. Now take (5), subtract $w(0)$ from both sides and plug $\omega(\sigma,p)$ on the right-hand side to get

$$w(\bar{y}) - w(0) = \left[\left(\frac{1}{p}(1+\omega(\sigma,p)^{-1})^{1-\sigma} - \frac{1-p}{p}\right)^{\frac{1}{1-\sigma}} - 1\right]\omega(\sigma,p)\epsilon, \tag{8}$$

which defines the term $\beta(\sigma,p)$. □